

2.1 Derivative Basics: Quiz E Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_quiz.py`). All problems are original; see the notes file for provenance. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Answer-only items** (Q1, Q3): all or nothing.
- **Value vs. derivative.** Giving a slope where a value was asked (or the reverse) earns 0 for that part.
- **Show-work items** (Q2, Q4, Q5): a bare correct answer earns at most half credit.
- **Definition items** (Q4(a), Q5): the problem states the required method. A correct answer found another way (for example, a power rule learned elsewhere) earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit.

Quiz E

Item	Answer	Rubric / trap
1(a)	-2	The numerator is reversed, so this is $-f'(2)$; the dashed tangent has slope 2: it passes through grid points two units apart, e.g. (1, 1) and (3, 5). <i>Trap</i> : 2, the slope itself: the numerator is $f(2) - f(x)$, reversed, so the limit is $-f'(2)$; earns 0.
1(b)	3	f is continuous at R , so the limit is the height $f(2) = 3$. <i>Trap</i> : the slope 2; earns 0.
1(c)	$y - 3 = 2(x - 2)$, i.e. $y = 2x - 1$	Point $R = (2, 3)$, slope 2. Any correct form. <i>Trap</i> : using the height as the slope, or $x + 2$ for $x - 2$; earns 0.
1(d)	P	Steepest uphill labeled point. Slopes at the labeled points: P : $\frac{7}{2}$, Q : -2 , R : 2, S : -3 , T : $-\frac{9}{2}$. <i>Trap</i> : R , the highest labeled point, or T , the steepest labeled point, which goes downhill; earns 0.
1(e)	Q, S, T	Points where the graph is going down (left to right). All or nothing. <i>Trap</i> : P, Q, T , the points below the x -axis.
2	$\frac{1}{5}$	1 pt $f(9) = 3$; 1 pt $f(4) = 2$; 2 pts $\frac{3 - (2)}{5} = \frac{1}{5}$. <i>Trap</i> : this is the slope of a secant line, not a limit. Letting $h \rightarrow 0$ gives $f'(4) = \frac{1}{4}$; earns 0.
3(a)	$c = -3$	$\frac{f(x) - f(-3)}{x - (-3)} = \frac{x^2 - 9}{x + 3}$; c is the number x approaches. (The limit is $f'(-3) = -6$, but the question asks for c .) <i>Trap</i> : $c = 3$, from $9 = 3^2$; earns 0. Giving the limit value -6 instead of c earns 0.
3(b)	-4	Factor: $\frac{(x - 1)(x + 1)(x^2 + 1)}{-(x - 1)} = -(x + 1)(x^2 + 1) \rightarrow -4$. <i>Trap</i> : 4, the slope of x^4 at 1; the denominator is $1 - x$, reversed; earns 0.

Item	Answer	Rubric / trap
4(a)	$f'(x) = \frac{1}{\sqrt{2x-1}}$	2 pts $f(x+h) = \sqrt{2(x+h)-1} = \sqrt{2x+2h-1}$; 2 pts multiplying top and bottom by the conjugate; 2 pts numerator $(2x+2h-1) - (2x-1) = 2h$, cancel h : $\frac{2}{\sqrt{2x+2h-1} + \sqrt{2x-1}}$; 2 pts letting $h \rightarrow 0$: $\frac{2}{2\sqrt{2x-1}}$. <i>Trap</i> : $f(x+h) = \sqrt{2x+h-1}$ (the 2 not distributed) leads to $\frac{1}{2\sqrt{2x-1}}$. Writing $f(x)+h$ for $f(x+h)$ earns at most 2.
4(b)	(5, 3)	2 pts solving $f'(x) = \frac{1}{\sqrt{2x-1}} = \frac{1}{3}$: $\sqrt{2x-1} = 3$, so $2x-1 = 9$ and $x = 5$; 2 pts the point (f evaluated there). Follow-through: full credit for correctly using the student's own $f'(x)$. <i>Trap</i> : giving only $x = 5$ earns 2.
5	$g'(-1) = \frac{3}{4}$	1 pt $g(-1) = -\frac{1}{2}$; 2 pts setup; 3 pts algebra: $\frac{\frac{x}{x+3} + \frac{1}{2}}{x+1} = \frac{2x + (x+3)}{2(x+3)(x+1)} = \frac{3(x+1)}{2(x+3)(x+1)} = \frac{3}{2(x+3)}$; 2 pts value at $x = -1$. <i>Trap</i> : $g(-1) = -\frac{1}{2}$, so subtracting it adds $\frac{1}{2}$; writing $\frac{x}{x+3} - \frac{1}{2}$ gives a numerator that does not cancel $x+1$. The denominator is $x+1$.

Points: $10 + 4 + 6 + 12 + 8 = 40$.